Merging of neutron star binaries in full general relativity

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Multiple Messengers and Challenges in Astroparticle Physics
L’Aquila, October 6 -17, 2014
Plan of the talk

- broad-brush picture of merging binary neutron stars
- role of the equation of state
- role of mass asymmetries
- role of magnetic fields
- towards a model of GRBs
why study binary neutron stars?

• We know they exist (as opposed to binary BHs) and are among the strongest sources of GWs

• We expect them related to SGRBs: energies released are huge: $10^{48-50}$ erg.

• No self-consistent model has yet been produced to explain them.

• Theoretical modelling has now reached level of maturity to shed light on central engine of SGRBs
The goals of numerical relativity

Numerical relativity solves Einstein equations in those regimes in which no approximation is expected to hold. We build codes which we consider as "theoretical laboratories".

\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi T_{\mu\nu} \quad \text{(field eqs: 6 + 6 + 3 + 1)} \]

\[ \nabla_\mu T^{\mu\nu} = 0, \quad \text{(cons. en./mom.: 3 + 1)} \]

\[ \nabla_\mu (\rho u^\mu) = 0, \quad \text{(cons. of baryon no: 1)} \]

\[ p = p(\rho, \epsilon, \ldots), \quad \text{(EoS: 1 + \ldots)} \]

\[ \nabla^* F^{\mu\nu} = 0, \quad \text{(Maxwell eqs.: induction, zero div.)} \]

\[ T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{em}} + \ldots \]

In non-vacuum space times the truncation error is the only measurable error: "SIMULATION"

It’s our approximation to "reality" and it can be continuously improved:

microphysics, magnetic fields, viscosity, radiation transport,...
The two-body problem in GR

• For BHs we know what to expect:
  
  \[ \text{BH} + \text{BH} \rightarrow \text{BH} + \text{gravitational waves (GWs)} \]

• For NSs the question is more subtle: the merger leads to an hyper-massive neutron star (HMNS), ie a metastable equilibrium:
  
  \[ \text{NS} + \text{NS} \rightarrow \text{HMNS} + \ldots \ ? \rightarrow \text{BH} + \text{torus} + \ldots \ ? \rightarrow \text{BH} \]

All complications are in the intermediate stages; the rewards high:
• studying the HMNS will show strong and precise imprint on the EOS
• studying the BH+torus should tell us on the central engine of GRBs

NOTE: with advanced detectors we expect to have a realistic rate of \(~40\text{ BNSs}\) inspirals a year, ie \(~1\) a week (Abadie+ 2010)
Hot EOS: high-mass binary $M = 1.6 M_\odot$

Animations: Kaehler, Giacomazzo, Rezzolla
Handling the inspiral

It’s the “cleanest” part of the problem: PN predicts point-particle dynamics + tidal corrections. Can we measure them?

Accuracy is expensive and clean convergence hard to reach.

Important recent progress: high-order accuracy with clean convergence (i.e. 3+) is possible for BNSs (Radice+ 2013a,b).
Handling the inspiral

Radice+ (2013a,b)

Computational saving best appreciated comparing phase error at the same resolution:
Whisky (order $\sim 1.8$)
WhiskyTHC (order $\sim 3.2$)

Cleaning convergence is essential for reliable results. Rarely figures of this type are shown for BNSs.
Handling the inspiral

We can distinguish what is **numerics** from what is **physics**. Yet, a **systematic** investigation is a computational challenge.
“merger → HMNS → BH + torus”

Quantitative differences are produced by:

- **differences induced by the gravitational **MASS:**
  a binary with smaller mass will produce a HMNS further away from the stability threshold and will collapse at a later time

- **differences induced by the **EOS:**
  a binary with an EOS with large thermal capacity (ie hotter after merger) will have more pressure support and collapse later
Dependence on EOS and mass is apparent. Perfect scenario much richer than in binary black holes.
realistic EOSs

With sufficiently sensitive detectors, GWs will work as the **Rosetta stone** to decipher the NS interior.
“merger \rightarrow HMNS \rightarrow BH + torus”

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- **differences induced by RADIATIVE PROCESSES:**
  radiative losses alter the equilibrium of HMNS; ejecta provide seeds for nucleosynthesis and radioactive decay
When considering realistic EOSs, two new degrees of freedom appear: temperature and electron fraction. Both are important in regulating the HMNS equilibrium and determining the multimessanger nature of BNSs.

SHT EOS, Shen et al. 2011

\[ M = 1.76 \, M_\odot \]

\[ R = 14.92 \, \text{km} \]

\[ d = 56.4 \, \text{km} \]
• On large scales, $T, \rho, Y_e$ do not track each other.

• About $< \sim 10^{-2} \, M_\odot$ are ejected; a fraction of this will undergo $r$-process nucleosynthesis.

• Other fraction will accrete back on the torus or onto the BH.
• About $\sim 10^{-2} \, M_\odot$ are ejected; exact amount depends on the EOS and mass ratio.

• Most of this material is unbound: permanent chemical pollution.

• Electron fraction is small, leading to rapid neutron-capture processes and nucleosynthesis.

• Yields reproduce all three peaks of cosmic abundance (Rosswog+ 2013, Perego+ 2014). BNSs alternative to SN.
neutrino radiative losses

• Introducing radiative losses consistently is a formidable challenge.
• The radiative transfer problem is intrinsically 7 dimensional and needs to be added to standard MHD/Einstein equations; out of reach of present supercomputers.
• Present approaches represent reasonable first approximations, “leakage scheme”

• In essence, neutrino act only as energy losses and are treated only in two regimes: diffusion and free-streaming.
• Overall, results indicate that luminosities of $10^{52}-10^{53}$ erg/s over HMNS phase, dropping of several orders after BH formation.
“merger → HMNS → BH + torus”

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  tidal disruption before merger; may lead to prompt BH
Total mass: $3.37 M_\odot$; mass ratio: $0.80$;

- the torii are generically more massive
- the torii are generically more extended
- the torii tend to stable quasi-Keplerian configurations
- overall unequal-mass systems have all the ingredients needed to create a GRB
“merger HMNS BH + torus”

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- **differences induced by MAGNETIC FIELDS:**
  the angular momentum redistribution via magnetic braking or MRI can increase/decrease time to collapse
Including magnetic fields: ideal MHD

NSs have large magnetic fields: that’s how we observe them. It is natural to ask:

• can we detect B-fields during the inspiral?
• how do B-fields influence the dynamics of the HMNS?
• how do B-fields influence the dynamics of the torus?

Not easy but can be done: ideal-MHD (infinite conductivity).

The B-fields are initially contained inside the stars

First interesting results in resistive-MHD; (Palenzuela+13, Dyonisopoulou+14)
B-fields during inspiral phase

Typical evolution for a magnetized binary (hot EOS) \( M = 1.5 \, M_\odot, B_0 = 10^{12} \, G \)

Animations: LR, Koppitz
Magnetic fields in the HMNS have complex topology: dipolar fields are destroyed.
Waveforms: comparing against magnetic fields

Compare B/no-B field:

- the evolution in the **inspiral** is different but only for ultra large B-fields (i.e. $B \sim 10^{17}$ G). For realistic fields the difference is not significant.

- the **post-merger** evolution is different for all masses; strong B-fields delay the collapse to BH.

However, **mismatch** must computed using detector sensitivity.
Can we detect B-fields in the inspiral?

To quantify the differences and determine whether detectors will see a difference in the inspiral, we calculate the overlap

$$\mathcal{O}[h_{B_1}, h_{B_2}] \equiv \frac{\langle h_{B_1} | h_{B_2} \rangle}{\sqrt{\langle h_{B_1} | h_{B_1} \rangle \langle h_{B_2} | h_{B_2} \rangle}}$$

where the scalar product is

$$\langle h_{B_1} | h_{B_2} \rangle \equiv 4\Re \int_0^\infty df \frac{\tilde{h}_{B_1}^* (f) \tilde{h}_{B_2} (f)}{S_h (f)}$$

In essence, at these res:

$$\mathcal{O}[h_{B_0}, h_B] \gtrsim 0.999$$

for \( B \lesssim 10^{17} \text{ G} \)

Because detectable mismatch is \( \lesssim 0.995 \), the influence of B-fields on the inspiral is unlikely to be detected.
B-fields during HMNS phase: MRI

Siegel et al, (2013)
While waiting for BH to be produced

- differentially rotating magnetized fluids can develop the magnetorotational instability or MRI (Velikhov 1959, Chandrasekhar 1960)
- the instability leads to an exponential growth of the magnetic field and to an outward transfer of angular momentum
- MRI responsible for accretion in most disc accretion scenarios
- An MRI developing in the HMNS has very important consequences:
  - ★ even small initial fields amplified of several orders of magnitude
  - ★ when BH is formed, torus material already highly magnetised
The challenges of global simulations

MRI in a nutshell (and a number of assumptions)

\[
\tau_{\text{MRI}} = \text{Im}(\omega_{\text{MRI}})^{-1} \sim \frac{1}{\Omega} \quad \lambda_{\text{MRI}} \sim \frac{2\pi}{\Omega} \frac{\mathbf{B} \cdot \mathbf{e}_k}{\sqrt{4\pi \rho}}
\]

• tipically \( \lambda_{\text{MRI}} \) is much smaller than typical size of astrophysical system, e.g., accretion discs, core-collapse supernovae, HMNS

• if unresolved, simulations cannot reproduce development of MRI

• so far the problem has been solved only with
  
  ✴ local simulations
  ✴ axisymmetry  \( \text{(Hawley & Balbus 92; Obergaulinger+ 06a,b, 09; Duez+ 06)} \)
First global simulations in full GR

Siegel + (2013)

- ideal MHD (WhiskyMHD code)
- ideal-fluid EOS, \( p = (\Gamma - 1) \rho \epsilon \)
- spacetime evolution (1+log slicing, Gamma-driver)
- axisymmetric initial model (\( M = 2.23M_\odot \))
  - purely poloidal B field (\( B_c^{\text{in}} = 5 \times 10^{17} \text{ G} \))
  - differential rotation: \( j \)-constant law
- cartesian grid \([0, 94.6] \times [0, 94.6] \times [0, 53.9] \text{ km}\)
- 4 refinement levels, finest gridspacing \( h = 44 \text{ m} \)
- \( \pi/2 \) and z-reflection symmetry
A local view in a global simulation

- Red box used for analysis has dimensions $(x, z) \in [1.0, 3.0] \times [1.0, 2.3] \text{ km}$
Magnetic field growth: linear and exponential

- **poloidal field** is not amplified during the evolution
- **toroidal field** initially generated by magnetic winding:

\[ B_{\text{tor}} \approx (rB^i \partial_i \Omega)t = a_w t \]
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\[\tau_{\text{MRI}} = (8.2 \pm 0.4) \times 10^{-2} \text{ ms}\]
Magnetic field growth: linear and exponential

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- at 0.28 ms MRI sets in with growth time:
  
  measured
  \[ \tau_{MRI, \text{fit}} = (8.2 \pm 0.4) \times 10^{-2} \text{ ms} \]

order-of-mag. prediction
\[ \tau_{MRI} \sim \Omega^{-1} \approx (4 - 5) \times 10^{-2} \text{ ms} \]

\( \tau_{MRI} \) does not depend on magnetic field strength
An important signature: channel flows

- onset of channel-flow merging visible in upper part
- power spectrum reveals a single dominant mode (apart from contributions from large-scale gradients)
- wavelength consistent with channel flows

\[ \lambda_{\text{MRI}} \approx 0.4 \text{ km} \]

order-of-mag. prediction

\[ \lambda_{\text{MRI}} \sim (0.5 - 1.0) \text{ km} \]

Altogether: first evidence for development of MRI in HMNSs
From a GW point of view, the binary becomes silent after BH formation and ringdown.

Is this really the end of the story?
Crashing neutron stars can make gamma-ray burst jets

\[ \frac{J}{M^2} = 0.83 \quad \text{ } \quad M_{\text{tor}} = 0.063M_\odot \quad \text{ } \quad t_{\text{accr}} \sim \frac{M_{\text{tor}}}{\dot{M}} \sim 0.3 \text{ s} \]
First time a *magnetic jet* is produced from *ab-initio* calculation: opening angle is $\sim 30^\circ$
how to extract information from the Rosetta stone

Takami, LR, Baiotti, 2014
Imprint of the EOS

APR4

$M = 1.275M_\odot$  $M = 1.300M_\odot$  $M = 1.325M_\odot$  $M = 1.350M_\odot$  $M = 1.375M_\odot$

ALF2

$M = 1.225M_\odot$  $M = 1.250M_\odot$  $M = 1.275M_\odot$  $M = 1.300M_\odot$  $M = 1.325M_\odot$

SLy

$M = 1.250M_\odot$  $M = 1.275M_\odot$  $M = 1.300M_\odot$  $M = 1.325M_\odot$  $M = 1.350M_\odot$

H4

$M = 1.250M_\odot$  $M = 1.275M_\odot$  $M = 1.300M_\odot$  $M = 1.325M_\odot$  $M = 1.350M_\odot$

GNH3

$M = 1.250M_\odot$  $M = 1.275M_\odot$  $M = 1.300M_\odot$  $M = 1.325M_\odot$  $M = 1.350M_\odot$

Takami, LR, Baiotti, 2014
Clearly, there are recurrent features!
A new approach to constrain the EOS

Takami, LR, Baiotti (2014)

We have carried out numerical-relativity simulations of NS binaries with nuclear EOS and thermal contribution via ideal-fluid contribution.
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PSD of post-merger GW signal has number of peaks (Oechslin+2007, Baiotti+2008)

The high-freq. peak ($f_2$) been studied carefully and produced by HMNS (Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013)

The low-freq. peak ($f_1$) is related to the early post-merger phase
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It is possible to correlate the values of the peaks with the properties of the progenitor stars, i.e. M, R, and combinations thereof.

Each cross refers to a given mass and crosses of the same color refer to the same EOS.

The high-freq. peak $f_2$ has been shown to correlate with stellar properties, e.g., $R_{\text{max}}$, $R_{1.6}$, etc (Bauswein+ 2011, 2012, Hotokezaka+ 2013).

The correlation depends on mass.

The low-freq. peak $f_1$ shows a much tighter correlation; most importantly, it does not depend on the EOS.
An example: start from equilibria

Assume that the GW signal from a binary NS is detected and with a SNR high enough that the two peaks are clearly measurable.

Consider your best choices as candidate EOSs
An example: use the $M(R, f_i)$ relation

The measure of the $f_i$ peak will fix a $M(R, f_i)$ relation and hence a single line in the $(M, R)$ plane.

All EOSs will have one constraint (crossing)
An example: use the $M(R, f_2)$ relations

The measure of the $f_2$ peak will fix a relation $M(R, f_2, \text{EOS})$ for each EOS and hence a number of lines in the $(M, R)$ plane.

The right EOS will have three different constraints (APR, GNH3, SLy excluded)
If the mass of the binary is measured from the inspiral, an additional constraint can be imposed.

The right EOS will have **four** different constraints. Ideally, a single detection would be sufficient.
An example: works for all EOSs considered

We have checked that the approach works well for all EOSs considered.
This works for all EOSs considered

In reality things will be more complicated. The lines will be stripes; Bayesian probability to get precision on $M, R$.

Some numbers:

- at 50 Mpc, freq. uncertainty from Fisher matrix is 100 Hz
- at SNR=2, the event rate is 0.2-2 yr$^{-1}$ for different EOSs.
Conclusions

✴ Modelling of binary NSs in full GR is mature: GWs from NSs are complex/rich can be the Rosetta stone to decipher the NS interior.

✴ Inspiral part is under control and mostly a computational challenge we are able to tackle.

✴ Magnetic fields unlikely to be detected during the inspiral but important after the merger: key to explain GRBs?

✴ First evidence of development of MRI in HMNSs opens important possibilities on the EM emission from this process.

✴ Essentially "universal" correlation between compactness and spectra of post-merger: a new approach to constrain EOS

BNSs are marvellous laboratories to study multi-messenger astroparticle physics. A lot more can be done!