First Part of Course (November-December): From Dynamics to Thermodynamics

One of the purposes of statistical mechanics is to explain thermodynamics in terms of the ‘microscopic’ dynamics. The general consensus is that thermodynamics describe the ‘macroscopic’ behavior of some quantities (energy, pressure, …) under the influence of external forces or heat bath. Macroscopic means that laws of thermodynamics emerge for ‘large’ systems and in a large time scale.

We present in these lectures a precise mathematical approach where thermodynamics laws (Carnot cycles, Kelvin and Clausius principles, etc., and the corresponding first and second principle of thermodynamics) are obtained through space-time scaling limits. In this approach the problem of thermodynamics is reduced to a purely mathematical problem, very difficult indeed.

We will avoid making a general theory, and we will work with the simplest system: a ‘one dimensional’ rubber subject to tension and heat bath. This will be modeled microscopically by a (classical) chain of oscillators (of Fermi-Pasta-Ulam type) subject to boundary tension forces and eventually stochastic Langevin heat bath.

Irreversible isothermal and adiabatic transformations can be obtained for the evolution of ‘local’ energy, momentum and volume stress, performing space-time scaling limits called ‘hydrodynamic limits’. Quasi-static reversible transformations, object of the classical thermodynamics, are then obtained in larger time scale.

If most of this mathematical program is still open, in particular concerning adiabatic transformations, some progress had been done if the Hamiltonian dynamics is perturbed by conservative noise (stochastic forces that conserve the total energy, density and eventually momentum).

No preliminary knowledge of thermodynamics or statistical mechanics is required, only some familiarity with certain elementary probability tools like sums of independent random variables and central limit theorems. Conditional distributions defining canonical and microcanonical Gibbs measures, and corresponding large deviations will be introduced in the course.
These are the chapters for the first part of the course (notes will be available):

- **the thermodynamics of a one-dimensional rubber**: temperature and equilibrium states, work energy and heat, adiabatic and isothermal transformations, quasi-static-transformations, Carnot cycles, Kelvin and Clausius principles and thermodynamic entropy. Extended system: local equilibrium.

**The statistical mechanics of the chain of oscillators**: Gibbs measures, thermodynamic entropy, equivalence of ensembles, local equilibrium.

**Hydrodynamic limits form the microscopic dynamics**: heat bath and isothermal diffusion, adiabatic hyperbolic evolution: Euler equations. Isothermal evolutions. Quasistatic limits.

Second Part of Course (January): Phase transitions in the two-dimensional Ising model.

**Ising model**: generalities, free energy, thermodynamic limit.

**Existence of Phase transitions at low temperature**: contours representation and Onsager argument.

**Uniqueness of phase at high temperature**: cluster expansion.