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Wavelets: basic constructions, theory and applications

A short preface

This is an introductory mini-course for graduate and PhD students. We begin with standard notions of functional analysis such as Hilbert space, orthonormal system, Fourier series and Fourier transform. We discuss several applications, where we see disadvantages of standard orthonormal bases and motivate their generalizations: Riesz bases, biorthogonal systems and Frames. Then we consider in detail the first wavelets: Haar system an its “dual” Shannon-Kotelnikov system, passing to more advanced Battle-Lemaire and Meyer systems, and finally, to compactly-supported wavelets, including Daubechies wavelets. The general construction of wavelets is introduced by the fundamental concept of multiresolution analysis, which motivates refinement functional equations as starting point. This allows us to derive the properties of compactly-supported wavelets, such as approximations, local and global regularity and relation to fractals. Finally we discuss applications to signal processing and numerical methods in PDE.

Program of the course

1. Hilbert spaces and their properties. The space $L_2(X)$. Orthonormal functional systems on a segment, on a line, on $\mathbb{R}^d$. Fourier series and Fourier transform. Time and frequency domains. Some applications of orthonormal systems (signal and image processing, numerical methods in PDE).


8. Three methods to estimate regularity of compactly-supported wavelets. The joint spectral radius. Wavelets and fractals: what is in common?


Literature